Time evolution of particle number asymmetry in an expanding universe

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Outline

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The motivation and the purpose

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- The asymmetry is one of problem in SM
- Some scenarios: baryogenesis[a], leptogenesis[b], etc.
- The origin of matter and anti-matter asymmetry of universe:
  - Mass difference of the fields
  - Interactions
  - Curvature (scale factor)

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The purpose

- Building an interacting model which generates particle number asymmetry in an expanding universe.
- Computing time evolution of asymmetry by using non-equilibrium quantum field theory.

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The Model

N: Neutral scalar, ϕ: Complex scalar

$$
\begin{aligned}
S &= \int d^4x \sqrt{-g}\left(\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int.}} + \xi(R - 2\Lambda)\right), \\
\mathcal{L}_{\text{free}} &= g^{\mu\nu}\nabla_\mu \phi^\dagger \nabla_\nu \phi - m_\phi^2 |\phi|^2 + \frac{1}{2}\nabla_\mu N \nabla^\mu N \\
&\quad - \frac{M_N^2}{2} N^2 + \underline{\frac{B^2}{2}(\phi^2 + \phi^{\dagger 2}) + \left(\frac{\alpha_2}{2}\phi^2 + h.c.\right) R} + \alpha_3 |\phi|^2 R \\
\mathcal{L}_{\text{int.}} &= \underline{A\phi^2 N + A^* \phi^{\dagger 2} N} + A_0 |\phi|^2 N
\end{aligned}
$$

A: interaction (vertex) coupling, B: giving the mass difference of fields, α_2: matter-curvature coupling

The interactions among them are CP violating and particle number violating.

U(1) transformation and initial condition

The particle number is related to U(1) transformation

- U(1) transformation of the complex scalar field $\phi'(x) \to \phi(x)e^{i\theta}$
- U(1) charge: particle-anti particle number[1]

$$N(x^0) = \int \sqrt{-g(x)} j_0(x) d^3\mathbf{x}$$

$$j_\mu(x) = i(\phi^\dagger \partial_\mu \phi - \partial_\mu \phi^\dagger \phi)$$

Initial condition: The state is given by density matrix

$$\rho(0) = \frac{e^{-\beta H_0}}{\mathrm{tr} e^{-\beta H_0}}, \quad \beta = \frac{1}{T}$$

The initial expectation value of scalar fields: $\bar{\phi}_i(0)$

[1] (I.Affleck and M.Dine, Nucl. Phys. B 249 (1985) 361)

The metric and Einstein equations

Space time: Friedmann Robertson Walker with scale factor $a(x^0)$,

$$g_{\mu\nu} = (1, -a^2(x^0), -a^2(x^0), -a^2(x^0))$$

$$R = 6\left[\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2\right], \;\; H(x^0) = \frac{\dot{a}}{a}$$

Einstein equations: We consider Einstein equation for scale factor coupled with scalar fields

$$\begin{aligned} \text{00component: } -3(1 - 8\pi G\beta_i\phi_i^2)\left(\frac{\dot{a}}{a}\right)^2 + \Lambda &= -8\pi G T_{00} \\ \text{ii component: } (1 - 8\pi G\beta_i\phi_i^2)(2a\ddot{a} + \dot{a}^2) - a^2\Lambda &= -8\pi G T_{ii} \\ \text{off diagonal component: } 0 &= -8\pi G T_{\mu\nu(\neq\mu)} \end{aligned}$$

$$T_{\mu\nu} = \partial_\mu\phi_i\partial_\nu\phi_i - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi_i\partial_\beta\phi_i - \frac{1}{2}m_i^2\phi_i^2 + \frac{1}{3}A_{ijk}\phi_i\phi_j\phi_k\right)$$

Complex scalar in term of real fields

One can decompose complex scalar → real and imaginary.

$$\phi \equiv \frac{\phi_1 + i\phi_2}{\sqrt{2}}, \quad \phi_3 \equiv N$$

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{3}\sqrt{-g}\left[\nabla_\mu\phi_i\nabla^\mu\phi_i - \bar{m}_i^2(x^0)\phi_i^2\right] + \phi_3\sum_{i,j=1}^{3}\phi_i A_{ij}$$

- The mass terms, B^2 and α_2, break U(1) symmetry, so that one complex scalar field splits into the two mass eigenstates of real scalars

$$\bar{m}_1^2(x^0) = m_\phi^2 - B^2 - (\alpha_2 + \alpha_3)R(x^0),$$
$$\bar{m}_2^2(x^0) = m_\phi^2 + B^2 + (\alpha_2 - \alpha_3)R(x^0).$$

Current expectation value in term of real fields

- The current:

$$j_\mu = \frac{1}{2}\phi_2 \overleftrightarrow{\partial}_\mu \phi_1 - \frac{1}{2}\phi_1 \overleftrightarrow{\partial}_\mu \phi_2$$

- Current expectation value with initial density matrix:

$$\begin{aligned}\langle j_\mu(x)\rangle &= \mathrm{tr}(j_\mu(x)\rho(0)) \\ &= \mathrm{Re.}\left(\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial y^\mu}\right) G_{12}(x,y)\Big|_{y\to x} + \mathrm{Re.}\left\{\bar{\phi}_2^*(x)\overleftrightarrow{\partial}_\mu \bar{\phi}_1(x)\right\}\end{aligned}$$

- $G_{ij}(x,y)$ and $\bar{\phi}_i$ are obtained from 2PI CTP EA.

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2PI formalism[2] (briefly)

The generating functional with source J and non-local source K is defined as,

$$e^{iW[J,K]} = \int d\Phi \text{Exp}\left\{ i\left[S + \int d^4x J_A(x)\Phi^A(x) + \frac{1}{2}\int d^4x d^4y \Phi^A(x) K_{AB}(x,y)\Phi^B(y)\right]\right\}$$

The sources are connected to the mean fields and Green's function through

$$\frac{\delta W}{\delta J_A} = \phi^A; \quad \frac{\delta W}{\delta K_{AB}} = \frac{1}{2}[\phi^A\phi^B + G^{AB}]$$

2PI CTP effective action (EA), Γ_2, is given by Legendre transform W

$$\Gamma_2[\phi, G] \quad = \quad W[J,K] - J_A\phi^A - \frac{1}{2}K_{AB}[\phi^A\phi^B + G^{AB}]$$

The equations of motion, i.e. Schwinger-Dyson equations, is given by

$$\frac{\delta \Gamma_2}{\delta \phi^A} = -J_A - K_{AB}\phi^B; \quad \frac{\delta \Gamma_2}{\delta G^{AB}} = -\frac{1}{2}K_{AB}$$

[2] E. Calzetta and B.L. Hu, Nonequilibrium Quantum Field Theory, Cambridge University, 2008

2PI CTP EA in curved space[3]

$$e^{iW[J,K]} = \int d\phi \exp\left(i\left[S + \int \sqrt{-g(x)}d^4x J_i^a c^{ab}\phi_i^b + \frac{1}{2}\int d^4x d^4y \sqrt{-g(x)}\phi_i^a(x) \right.\right.$$
$$\left.\left. \times c^{ab}K_{ij}^{bc}(x,y)c^{cd}\phi_j^d(y)\sqrt{-g(y)}\right]\right)$$

$$\Gamma_2[G,\bar{\phi},g] = S[\bar{\phi},g] + \frac{i}{2}\mathrm{TrLn}\, G^{-1} + \Gamma_Q - \frac{i}{2}\mathrm{Tr}\,\mathbf{1}$$
$$+\frac{1}{2}\int d^4x \int d^4y \frac{\delta^2 S[\bar{\phi},g]}{\delta\bar{\phi}_i^a(x)\delta\bar{\phi}_j^b(y)} G_{ij}^{ab}(x,y),$$

- Solving Schwinger-Dyson equations for field and GF

$$\frac{\delta\Gamma_2}{\delta\bar{\phi}^A} \text{ and } \frac{\delta\Gamma_2}{\delta G^{AB}}$$

- Input the solution to the current expression

$$\langle j_\mu(x)\rangle = \mathrm{Re.}\left(\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial y^\mu}\right) G_{12}(x,y)\Big|_{y\to x} + \mathrm{Re.}\left\{\bar{\phi}_2^*(x)\overleftrightarrow{\partial}_\mu\bar{\phi}_1(x)\right\}$$

- Non-zero current ⇔ particle number violation (asymmetry)

[3] S.A. Ramsey and B.L. Hu, Phys. Rev. D 56, 661 (1997)

Rescaling Green's function and field

$$
\begin{aligned}
\bar{\phi}(x^0) &= \left(\frac{a_0}{a(x^0)}\right)^{3/2} \hat{\varphi}(x^0) \\
G(x^0, y^0, \mathbf{k}) &= \left(\frac{a_0}{a(x^0)}\right)^{3/2} \hat{G}(x^0, y^0, \mathbf{k}) \left(\frac{a_0}{a(y^0)}\right)^{3/2} \\
\hat{A}(x^0) &= \left(\frac{a_0}{a(x^0)}\right)^{3/2} A
\end{aligned}
$$

- The solutions are written in term of integral equations which are iteratively solved by treating interaction coupling A is small.
- We are interested the solutions up to first order of A.

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$$\left(\frac{a(x^0)}{a_0}\right)^3 \langle j_0(x^0)\rangle_{o(A)} = \int \frac{d^3k}{(2\pi)^3}\left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial y^0}\right)[\mathrm{Re}.\hat{G}_{12,\mathrm{int}}(x^0, y^0, \mathbf{k})]\big|_{y^0\to x^0}$$
$$+\mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{free}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{free}}(x^0)\}+\mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{free}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{int}}(x^0)\}$$
$$+\mathrm{Re}\{\hat{\varphi}^*_{2,\mathrm{int}}(x^0)\overleftrightarrow{\partial}_\mu\hat{\varphi}_{1,\mathrm{free}}(x^0)\}$$

$$B \to 0 \to \omega_{1,\mathbf{0}} = \omega_{2,\mathbf{0}}, \bar{K}_2 = \bar{K}_1; \quad \bar{K}_{i,x^0y^0,\mathbf{k}} = \frac{\sin\omega_{i,\mathbf{k}}(x^0-y^0)}{\omega_{i,\mathbf{k}}}$$
$$B \neq 0 \to \omega_{1,\mathbf{0}} \neq \omega_{2,\mathbf{0}}, \bar{K}_2 \neq \bar{K}_1$$

Parameters	j_0^{free}	j_0^{GF}	$j_0^{\mathrm{Free.Int}}$	$j_0^{\mathrm{Free.Int}}$
B	$B \neq 0$	$B \neq 0$	$B \neq 0$ or $B = 0$	$B \neq 0$ or $B = 0$
A	-	$A_{123} \neq 0$	$A_{123} \neq 0$	$A_{113} \neq A_{223}$
$\hat{\varphi}_{i,0}$	$\hat{\varphi}_{1,0} \neq 0$, $\hat{\varphi}_{2,0} \neq 0$	$\hat{\varphi}_{3,0} \neq 0$	$\hat{\varphi}_{3,0} \neq 0$, $\hat{\varphi}^2_{1,0} \neq \hat{\varphi}^2_{2,0}$	$\hat{\varphi}_{1,0}\hat{\varphi}_{2,0}\hat{\varphi}_{3,0} \neq 0$

Table 1: All requirements for non zero current

Time dependence asymmetry and limit for flat limit case

- The asymmetry is generated with U(1) symmetric initial condition for expectation value of fields.
- The non-zero asymmetry in the vanishing limit $\hat{\varphi}_{1,0} = \hat{\varphi}_{2,0} = 0$ comes from $O(A)$ contribution to the Green function.
- In the limit $H \to 0$ (flat case), GF contribution:

$$\lim_{H\to 0} \langle j_0(x^0) \rangle_{o(A^1)} = \int \frac{d^3k}{(2\pi)^3} \left(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial y^0} \right) [\mathrm{Re}.\hat{G}_{12,\mathrm{int}}(x^0, y^0, \mathbf{k})]\Big|_{y^0 \to x^0}$$

- Though the asymmetry includes a divergent term, the dimensional regularization leads to the finite result.

The absence of the ultraviolet divergence

- We study the ultraviolet divergence of the asymmetry due to the spacial momentum integration d^3k.
- The term which is naively divergent is identified.

$$\int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_1(k)\omega_2(k)} \frac{\frac{\sin \omega_3 x^0}{\omega_3} - \frac{\sin(\omega_1(k)-\omega_2(k))x^0}{\omega_1(k)-\omega_2(k)}}{\omega_3^2 - (\omega_1(k) - \omega_2(k))^2} \Bigg|_{UV.}^{div.}$$

$$\sim \quad \frac{1}{\omega_3^2}\left(\frac{\sin \omega_3 x^0}{\omega_3} - x^0\right) \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_1(k)\omega_2(k)}$$

$$\lim_{k\to\infty} (\omega_1(k) - \omega_2(k)) \to \frac{m_1^2 - m_2^2}{2k} \to 0.$$

Dimensional regularization

Dimensional regularization leads to finite result

$$
\begin{aligned}
& \lim_{d\to 4}\int \frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{\omega_1(k)\omega_2(k)} \\
= & \lim_{d\to 4}\frac{1}{2}\int \frac{d^{d-1}k}{(2\pi)^{d-1}}\left(\frac{1}{k^2+m_1^2}+\frac{1}{k^2+m_2^2}\right) \\
- & \lim_{d\to 4}\int \frac{1}{2}\frac{d^{d-1}k}{(2\pi)^{d-1}}\left(\frac{\omega_2(k)-\omega_1(k)}{\omega_1(k)\omega_2(k)}\right)^2 \\
= & -\frac{m_1+m_2}{8\pi}+\text{finite integral.}
\end{aligned}
$$

$$
\lim_{d\to 4}\int \frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{k^2+m^2} = -\frac{m}{4\pi}
$$

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Numerical calculation (work in progress)

Here we choose $m_1 = 2$, $m_2 = 3$, $\omega_3 = 0.35$ and $T = 100$

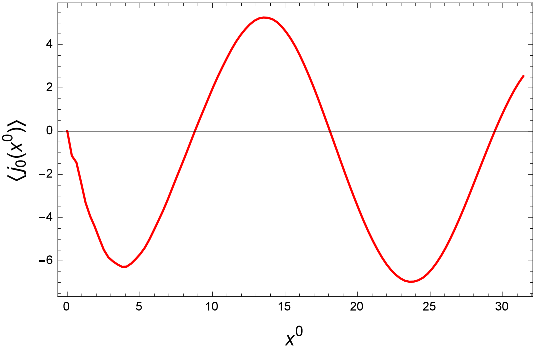

Figure 1: Time dependence for current

In the limit for non-zero asymmetry, the period of asymmetry oscillation is much longer than the age of universe.

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Summary and future research

Summary

- In an interacting model, particle number asymmetry can be generated through interactions of scalar fields
- The current for the particle and anti-particle asymmetry is given up to the first order of A. The asymmetry up to $O(A)$ is finite
- Non-vanishing particle number asymmetry depends on some parameters, such as temperature, small coupling A, ω_3, mass difference, and the initial value of scalar particle $\phi_i(0)$.

Future research

- Numerical calculation of the asymmetry should be carried out by changing the parameters of the model $(m_1, m_2, \omega_3,)$ and T.
- Calculation for arbitrary scale factor
- Solving Einstein equation and Schwinger-Dyson equation simultaneously (or sequentially)
- Possibility to identify our scalar fields as inflaton, dilaton, Higgs or dark matter etc.

THANK YOU...