Revisiting Yang-Mills thermodynamics: role of the Polyakov loop

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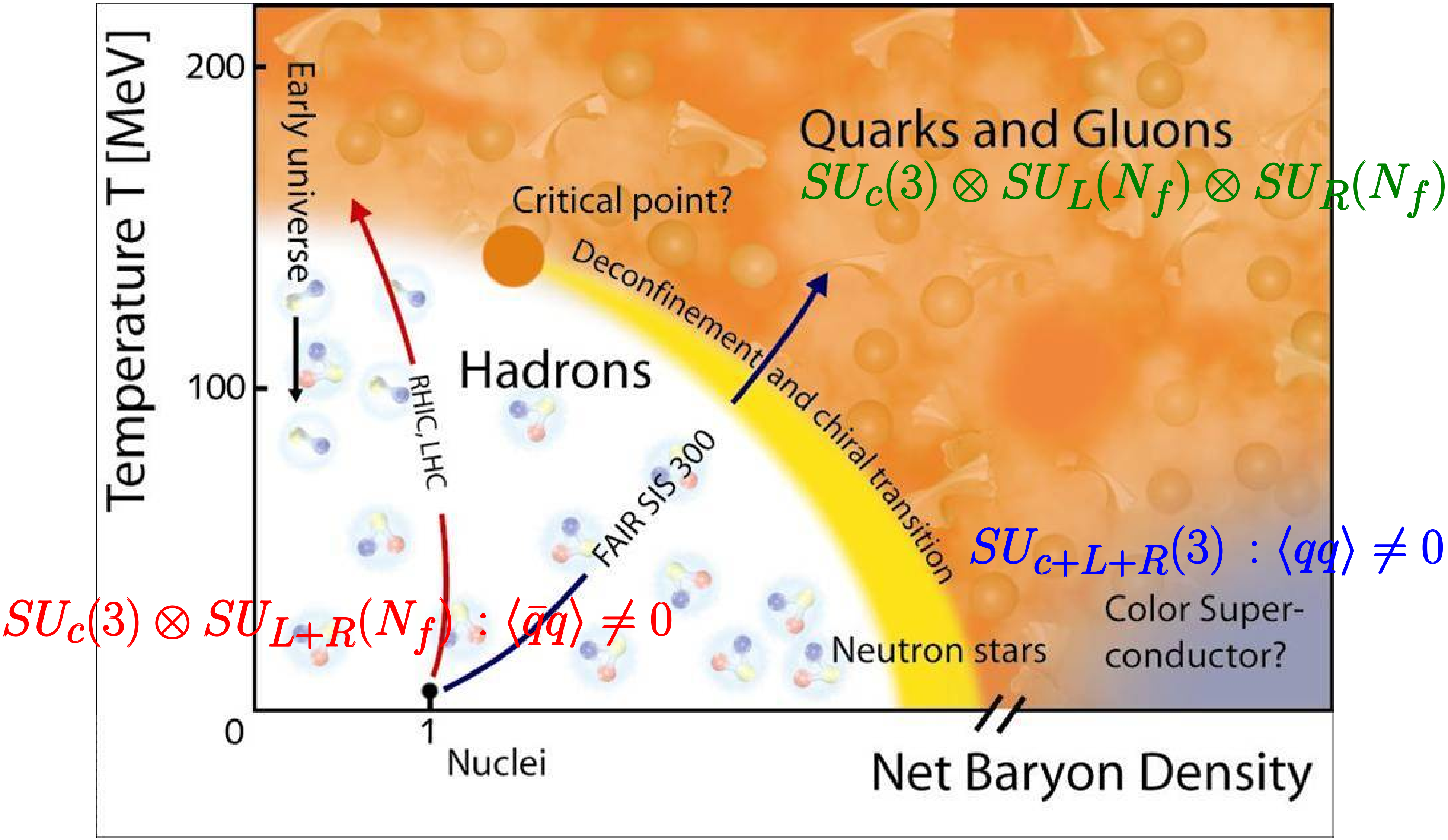
based on:
C.S. and K. Redlich, Phys. Rev. D **86**, 014007 (2012).

Why QCD matter under extreme conditions?

- **the origin of hadron masses**
  - disentangle the mechanism of the mass generation by reversing the process! $\Rightarrow$ dynamical "disappearance" of the mass
  - **heated**: going backward in time in the evolution of **the Universe**
  - **compressed**: several times of the nuclear matter density

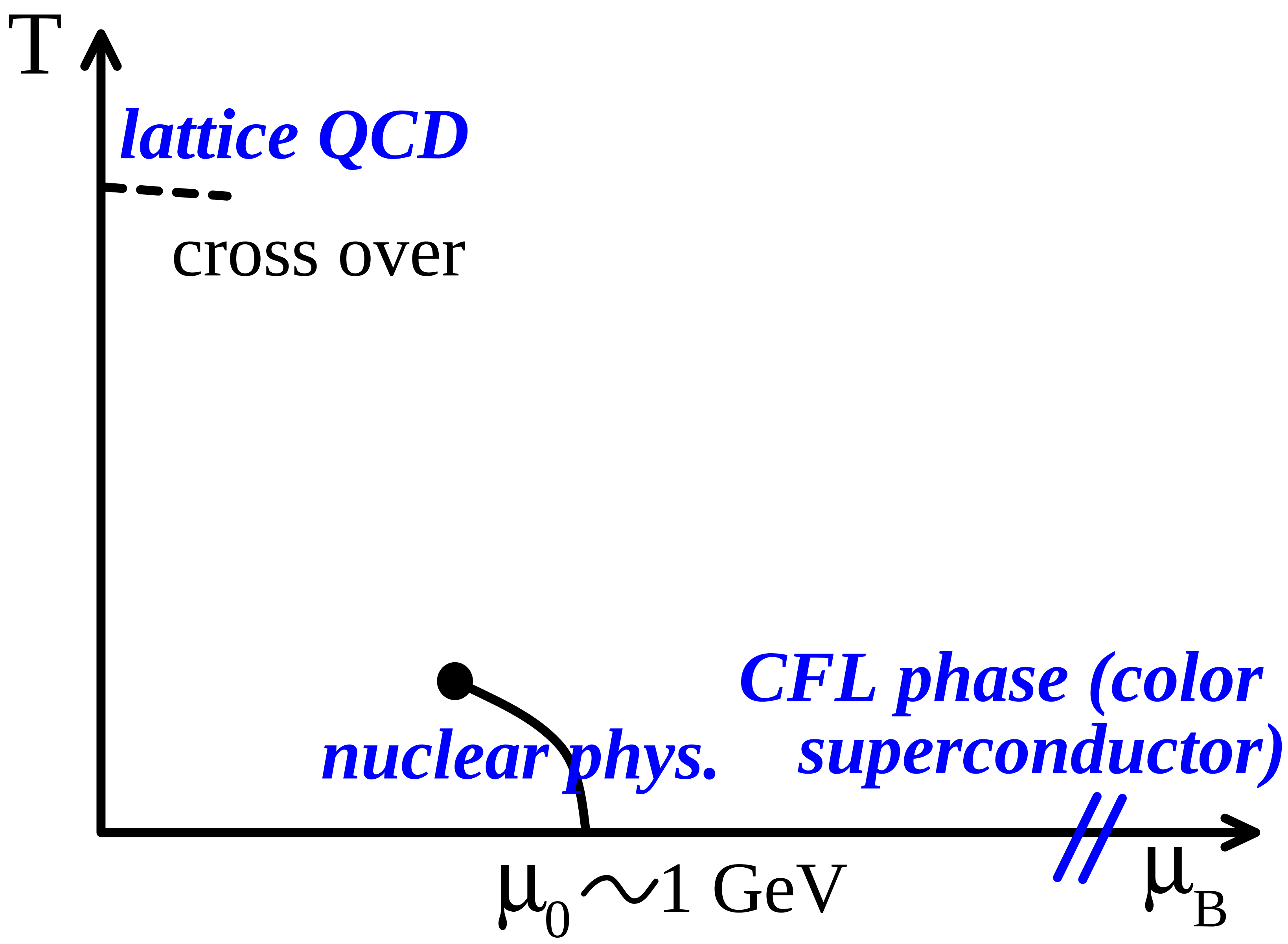
    $\sim$ **interior of compact stars**
- **interplay between chiral dynamics and color confinement**
  - Lattice QCD at small μ/T: no true phase transition but **crossover**
  - which takes place first? at $\mu \neq 0$ LQCD is not accessible.
- **compact stars and EoS**
  - hadronic vs. quark matter in their interior? — hadron physics territory!
- **accessible in current and future experiments**
  - hot matter: RHIC (BNL), SPS, LHC (CERN)
  - dense matter: FAIR (GSI), NICA (Dubna), J-PARC (Tokai)

- **the QCD phase diagram...?**



from http://www.gsi.de/fair/overview/research/

- … but actually what we know is



Outline

- Introduction: modeling YM/QCD thermodynamics
  - quasi-particle approach
  - Nambu–Jona-Lasinio/quark-meson models with Polyakov loops
- How do gluons and Polyakov-loops live together?
- A hybrid approach: dilatons as glueballs
- Issues:
  - non-perturbative effect above T_c:
    - interaction measure, effective gluon mass
    - electric vs. magnetic gluons $\Leftrightarrow$ Polyakov loops vs. dilatons
  - introducing fermions

I. Introduction

Model approaches for QCD thermodynamics

- **quasi-particle model** [Peshier et al., (96), Levai-Heinz (97)]
  - "dressed" gluons and quarks following the dispersion relation:

$$\omega^2 = \vec{k}^2 + m^2(T)$$

entropy density (ideal-gas form):

$$s = \frac{d}{2\pi^2} \int_0^\infty dk f_B \frac{k^2}{E} \left(\frac{4}{3} k^2 + m^2 \right)$$

pressure and energy density:

$$p = \frac{d}{6\pi^2} \int_0^\infty dk f_B \frac{k^4}{E} - B(T) \,,$$

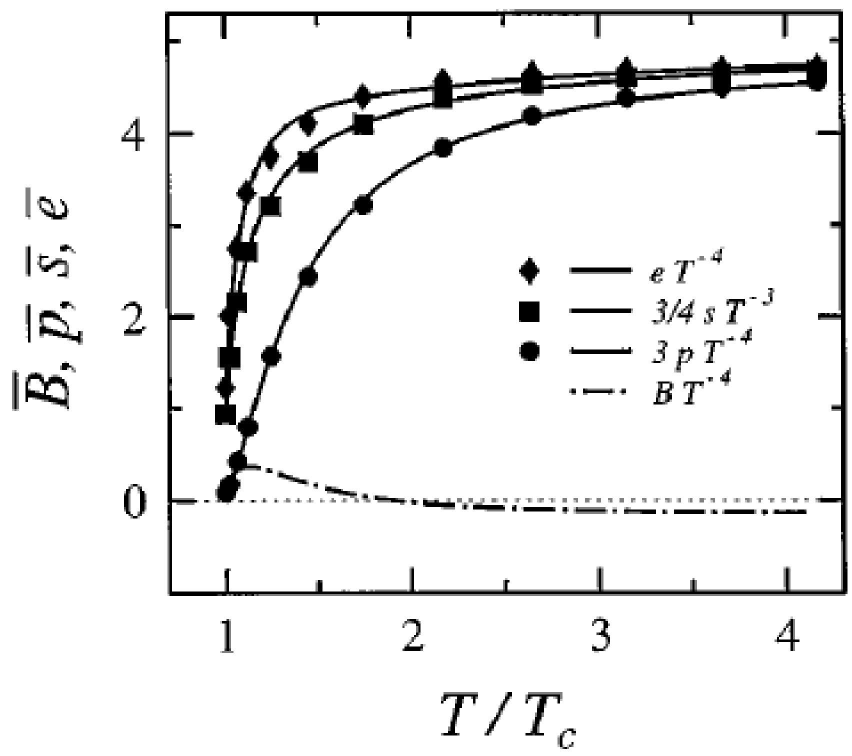
$$\epsilon = \frac{d}{2\pi^2} \int_0^\infty dk f_B k^2 E + B(T) \,.$$

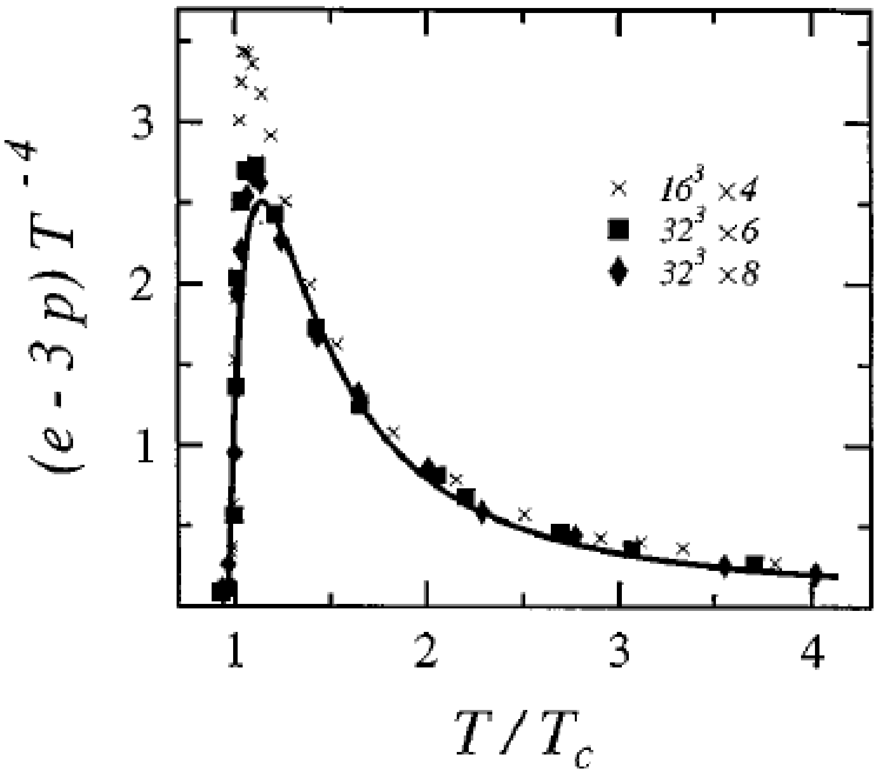
thus thermodynamic law $\epsilon + p = Ts$ fulfilled.

$$B(T) = B_0 - \frac{d}{4\pi^2} \int_{T_0}^T d\tau \frac{dm^2(\tau)}{d\tau} \int_0^\infty \frac{dk k^2 f_B}{E} \,.$$

$$m^2(T) = \frac{N_c}{6} G^2(T) T^2 \,, \quad G(T) \sim 1/\ln{(T/T_c)} \ .$$

– fitting lattice EOS





– confinement?

- **Polyakov-loop model** [Pisarski (00)]
  - $Z(N_c)$ symmetries

$$\Omega_c = e^{i\phi}\mathbf{1}\,, \quad \text{and} \quad \det\Omega_c = 1$$

$$\phi = \frac{2\pi j}{N_c}\,, \quad j = 0, 1, \cdots, (N_c - 1) : Z(N_c)\,\text{symmetry}$$

Note: quark breaks $Z(N_c)$ explicitly.

  - Polyakov loop ($A_4 = iA_0$)

$$\hat{L}(\vec{x}) = \mathcal{P}\exp\left[i\int_0^{1/T} d\tau A_4(\vec{x},\tau)\right]$$

$$\Phi = \mathrm{Tr}\hat{L}/N_c\,, \quad \bar{\Phi} = \mathrm{Tr}\hat{L}^\dagger/N_c$$

$*\ \Phi \to e^{i\frac{2\pi j}{N_c}}\Phi$: its VEV is an order parameter of $Z(N_c)$ breaking

$*$ heavy-quark potential [McLerran-Svetitsky (81)]

$$\langle\Phi(\vec{x})\rangle \sim e^{-F_q(\vec{x})/T} \begin{cases} = 0 & \text{confined phase} \\ \neq 0 & \text{deconfined phase} \end{cases}$$

  - effective potential: $\dfrac{\mathcal{U}}{T^4} = a\bar{\Phi}\Phi + b(\bar{\Phi}^3 + \Phi^3) + c(\bar{\Phi}\Phi)^2 + \cdots$

- **NJL with Polyakov loops** [Meisinger-Ogilvie (96), Fukushima (03), Ratti et al. (06)]
  - embedding Φ in a chiral framework

$$\mathcal{L} = \bar{q}\,(i\partial\!\!\!/ - A\!\!\!/)\,q + G\,(\bar{q}q)^2 - \mathcal{U}(\bar{\Phi}, \Phi)\,, \quad A_\mu = \delta_{\mu 0} A^0$$

  - constant background $A_0 \Rightarrow$ "chemical potential"

$$\mu \longrightarrow \mu + iA_4$$

$$\Omega_q = -d_q T \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr} \ln \left[1 + \hat{L}_F e^{-(E_q-\mu)/T}\right]$$

$$\hat{L}_F = \mathrm{diag}\left(e^{i\phi_1}, e^{i\phi_2}, e^{-i(\phi_1+\phi_2)}\right)$$

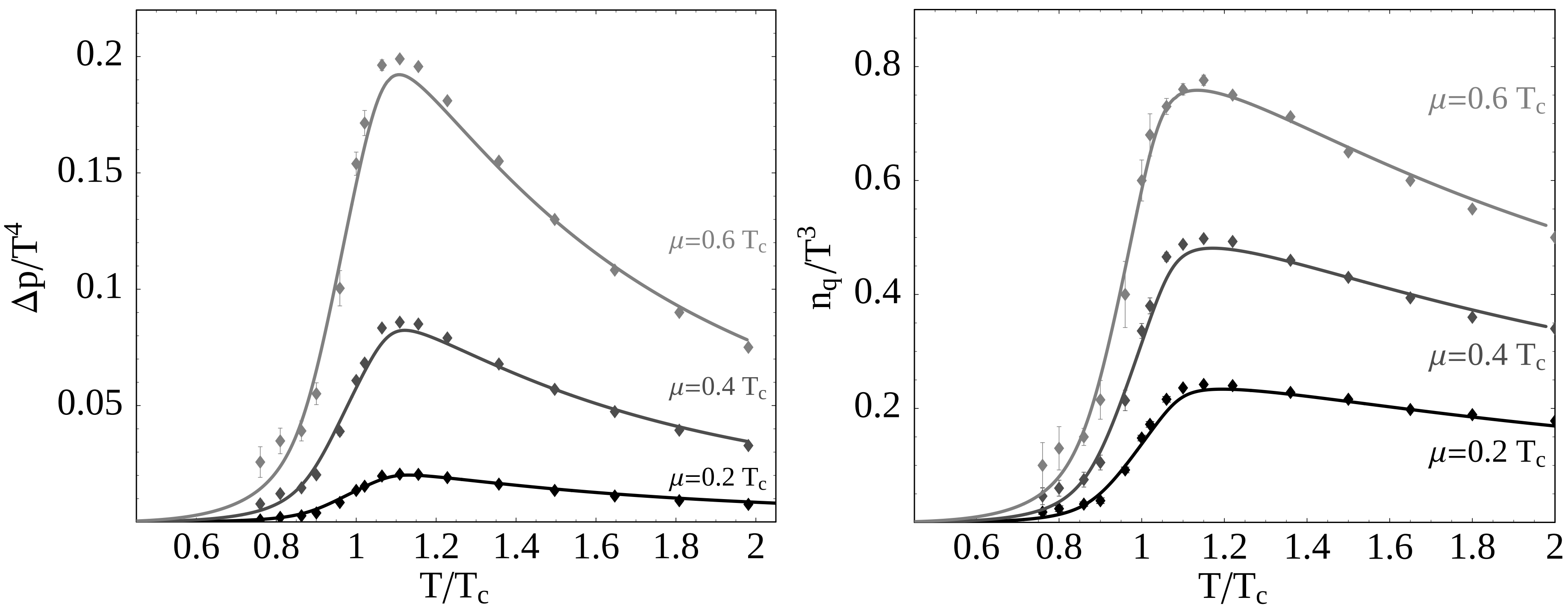
thus

$$\Omega_q = -d_q\,T \int \frac{d^3p}{(2\pi)^3} \ln \left[1 + 3\Phi e^{-E_+/T} + 3\bar{\Phi} e^{-2E_+/T} + e^{-3E_+/T}\right]$$

  - full thermodynamic potential

$$\Omega = \Omega_{q+\bar{q}}(M, \Phi, \bar{\Phi}; T, \mu) + V_{4\mathrm{F}}(M) + \mathcal{U}(\Phi, \bar{\Phi}; T)$$

[Ratti et al. (06)]



"Confinement" in PNJL/PQM models

- NJL/QM under a constant background A_0

[Meisinger-Ogilvie (96), Fukushima (03), Ratti-Thaler-Weise (06)]

$$\mathcal{L}_{\rm kin} = \bar{q}\,(i\partial\!\!\!/ - A\!\!\!/_0)\,q$$
$$\Rightarrow\ \Omega_q = d_q\,T\int\frac{d^3p}{(2\pi)^3}\ln\left[1+3\Phi e^{-E/T}+3\Phi e^{-2E/T}+e^{-3E/T}\right]$$

$\langle\Phi\rangle\simeq 0$ at low T: 1- and 2-quark states thermodynamically irrelevant
$\Rightarrow$ mimicking confinement

- **pure gauge sector** of PNJL/PQM: $\Omega_g = T^4\mathcal{U}(\Phi;T)$
  - made based on $Z(3)$ symmetry:

$$\mathcal{U} = a(T)\bar{\Phi}\Phi + b(T)\left(\bar{\Phi}^3+\Phi^3\right) + c(T)\left(\bar{\Phi}\Phi\right)^2 + \cdots$$

  - T-dep. of coefficients $\Leftrightarrow$ Lattice EoS **"bottom-up"**
  - Polyakov-loop susceptibility from LQCD [Karsch-Laermann (94), Allton et al. (02)]
    $\Rightarrow$ insufficient $\mathcal{U}$ for fluctuations [CS-Friman-Redlich (06)]
  - where T-dep. comes from? $\cdots$ thermal gluon excitations $\Leftrightarrow \mathcal{L}_{\rm YM}$

- revisit pure SU(3) YM theory
  closer contact with the underlying theory **"top-down"**
  $\Rightarrow$ better low-energy effective theory, thus better thermodynamics

Q. How can gluons and Φ live together?
Gluon thermodynamics in low-T phase?
Similar mechanism to quark sector?

II. Effective Gluon Potential

Deriving partition function from YM Lagrangian

- background field method, a constant uniform background $\bar{A}_0$

$$A_\mu = \bar{A}_\mu + g\check{A}_\mu$$
$$\bar{A}^a_\mu = \bar{A}^a_0 \delta_{\mu 0} \,, \quad \bar{A}_0 = \bar{A}^3_0 T^3 + \bar{A}^8_0 T^8$$

$$\sum_n \ln\det\left(D^{-1}\right) = \ln\det\left(1 - \hat{L}_A e^{-|\vec{p}|/T}\right)$$

[Gross-Pisarski-Yaffe (81)]

- Polyakov loop matrix in adjoint representation (8x8 matrix)

$$\hat{L}_A = \text{diag}\left(1\,,1\,,e^{i(\phi_1-\phi_2)}\,,e^{-i(\phi_1-\phi_2)}\,,e^{i(2\phi_1+\phi_2)}\,,e^{-i(2\phi_1+\phi_2)}\,,e^{i(\phi_1+2\phi_2)}\,,e^{-i(\phi_1+2\phi_2)}\right)$$

rank of SU(3) group $= 2$ $\Rightarrow$ elements expressed in 2 variables

- express in terms of

$$\Phi = \text{tr}\hat{L}_F/3\,,\ \bar{\Phi} = \text{tr}\hat{L}^\dagger_F/3$$

- full thermodynamics potential:

$$\Omega = \underbrace{\Omega_g}_{\sim a(T)\bar{\Phi}\Phi?} + \underbrace{\Omega_{\text{Haar}}}_{\text{responsible for Z(3) breaking}}$$

- Haar measure: Jacobian associated with $L \Leftrightarrow A_0$

$$M(\phi_1,\phi_2) = \frac{8}{9\pi^2}\sin^2\left(\frac{\phi_1-\phi_2}{2}\right)\sin^2\left(\frac{2\phi_1+\phi_2}{2}\right) \times \sin^2\left(\frac{\phi_1+2\phi_2}{2}\right),$$

normalization:

$$\int_0^{2\pi}\int_0^{2\pi} d\phi_1 d\phi_2 M(\phi_1,\phi_2) = 1$$

partition function:

$$Z = \int d\phi_1 d\phi_2 M(\phi_1,\phi_2) e^{S(\phi_1,\phi_2)} = \int d\phi_1 d\phi_2 e^{S_{\text{eff}}(\phi_1,\phi_2)},$$

$$S_{\text{eff}} = S(\phi_1,\phi_2) + \ln M(\phi_1,\phi_2)$$

$$M(\phi_1,\phi_2) = \frac{8}{9\pi^2}\left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3+\bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right]$$

$\Rightarrow$ restricts the (ϕ_1,ϕ_2) target space

$\Rightarrow$ Φ never exceeds unity.

- full thermodynamics potential: $\Omega = \Omega_g + \Omega_{\mathrm{Haar}}$

$$\Omega_g = 2T \int \frac{d^3p}{(2\pi)^3} \ln \left(1 + \sum_{n=1}^{8} C_n(\Phi, \bar{\Phi})\, e^{-n|\vec{p}|/T} \right) ,$$

$$\Omega_{\mathrm{Haar}} = -a_0 T \ln \left[1 - 6\bar{\Phi}\Phi + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2 \right] ,$$

$$C_1 = C_7 = 1 - 9\bar{\Phi}\Phi\,, \quad C_2 = C_6 = 1 - 27\bar{\Phi}\Phi + 27\left(\bar{\Phi}^3 + \Phi^3\right) ,$$

$$C_3 = C_5 = -2 + 27\bar{\Phi}\Phi - 81\left(\bar{\Phi}\Phi\right)^2 ,$$

$$C_4 = 2\left[-1 + 9\bar{\Phi}\Phi - 27\left(\bar{\Phi}^3 + \Phi^3\right) + 81\left(\bar{\Phi}\Phi\right)^2 \right] , \quad C_8 = 1$$

$\Rightarrow$ energy distributions solely determined by group characters of SU(3)

– no free parameter in Ω_g

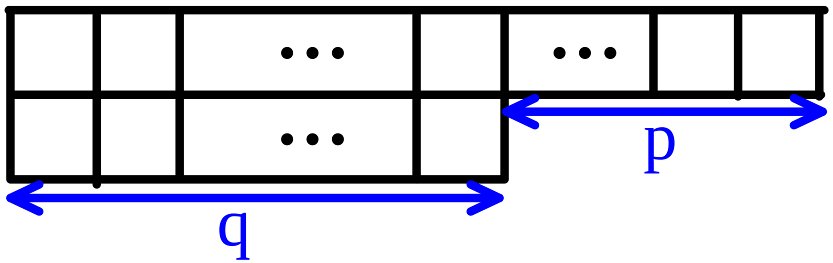
– one parameter in Ω_{Haar}: $a_0 \Leftrightarrow T_c^{\mathrm{lat}} = 270$ MeV

Character expansion of Ω_g

- effective action in the strong coupling exp. [Wozar-Kaestner-Wipf-Heinzl-Pozsgay (06)]

$$\mathcal{S}_{\rm eff}^{\rm (SC)} = \lambda_{10} S_{10} + \lambda_{20} S_{20} + \lambda_{11} S_{11} + \lambda_{21} S_{21}$$

S_{pq}: products of SU(3) characters $\sim$ a series of $Z(3)$-inv. operators



$$C_{1,7} = S_{10}\,, \quad C_{2,6} = S_{21}\,,$$
$$C_{3,5} = S_{11}\,, \quad C_4 = S_{20}$$

- a "minimal" model: $\mathcal{S}_{\rm eff} = \lambda S_{10} \sim \lambda \bar{\Phi}\Phi$ plus $\mathcal{S}_{\rm Haar}$
  $\Rightarrow$ 1st-order phase transition

- coefficient λ can be deduced from Ω_g! $\quad \Omega_g \simeq \mathcal{F}(T)\bar{\Phi}\Phi$

- cf. "phenomenological" potentials used in PNJL/PQM
  $\Omega = a(T) T^4 \bar{\Phi}\Phi + \Omega_{\rm Haar}$: unknown $a(T)$ fixed by fitting Lattice EoS

III. Thermodynamics

Thermodynamics

- high temperature limit: $\Phi \to 1 \Rightarrow$ non-int. gluon gas

$$\Omega_g(\Phi = \bar{\Phi} = 1) = 16T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 - e^{-|\vec{p}|/T}\right)$$

- any finite temperature in confined phase: $\Phi = 0$ thus $\Omega_{\text{Haar}} = 0$

$$\Omega_g(\Phi = \bar{\Phi} = 0) \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-|\vec{p}|/T}\right)$$

wrong sign! $\Rightarrow$ unphysical EoS $s, \epsilon < 0$

Gluons are NOT correct dynamical variables below T_c!

cf. PNJL/PQM: quarks are suppressed but exist at any T.

- **higher representations of Polyakov loop**
  - non-vanishing in confined phase *within mean field approx.*
  - do not condense when energy distributions are expressed in fund. rep.

$\Rightarrow$ the correct physics restored!

A hybrid approach for Yang-Mills thermodynamics

- below T_c: no gluons but **glueballs**
  $\Rightarrow$ introduce scalar glueballs as dilatons χ
- **QCD trace anomaly**:
  $T^\mu_\mu \sim \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$ scale symmetry breaking due to gluon condensate
  $\Rightarrow$ this is encoded with dilaton potential such that $T^\mu_\mu \sim \chi^4$ [Schechter (80)]

$$V_\chi = \frac{B}{4} \left(\frac{\chi}{\chi_0} \right)^4 \left[\ln \left(\frac{\chi}{\chi_0} \right)^4 - 1 \right]$$

- thermodynamics of glueballs

$$\Omega = \Omega_\chi + V_\chi + \frac{B}{4} ,$$

$$\Omega_\chi = T \int \frac{d^3p}{(2\pi)^3} \ln \left(1 - e^{-E_\chi/T} \right) ,$$

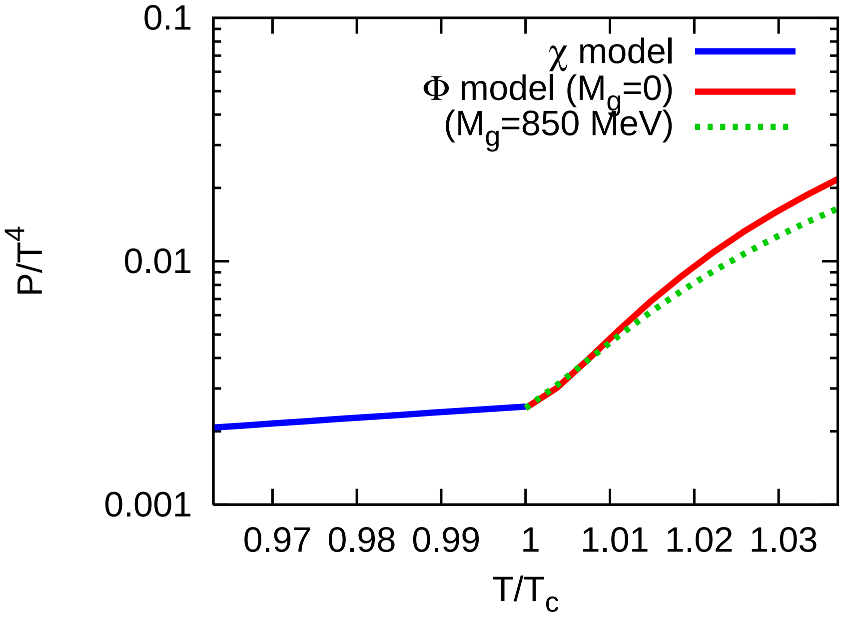
$$E_\chi = \sqrt{|\vec{p}|^2 + M_\chi^2} , \quad M_\chi^2 = \frac{\partial^2 V_\chi}{\partial \chi^2} ,$$

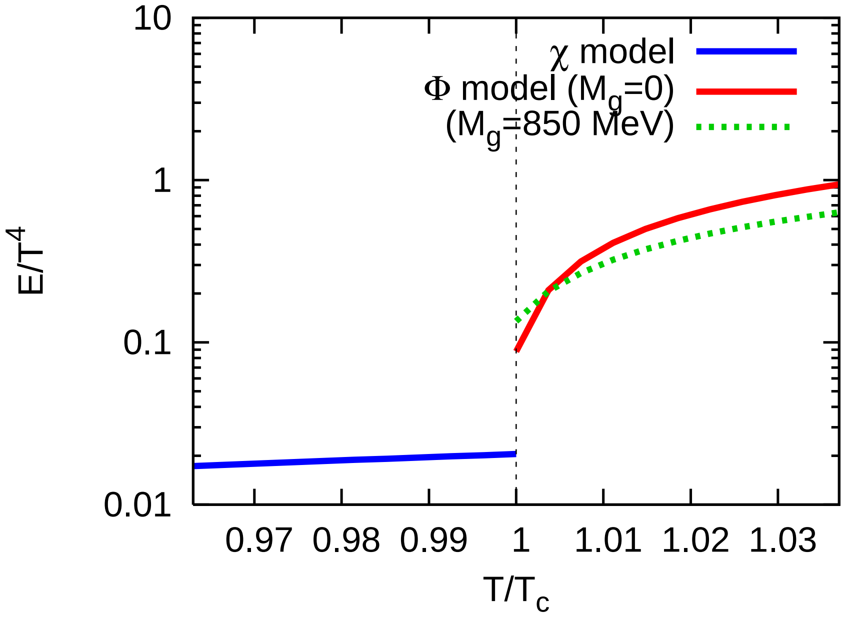
- below T_c: no gluons but glueballs
  $\Rightarrow$ switching dynamical variables at T_c

$$\Omega = \Theta(T_c - T)\,\Omega(\chi) + \Theta(T - T_c)\,\Omega(\Phi)\,,$$
$$\Omega(\chi) = \Omega_\chi + V_\chi + B/4\,, \quad \Omega(\Phi) = \Omega_g + \Omega_{\rm Haar} + c_0$$

- $\epsilon_0 = 0.6$ GeV fm^{-3}, $M_G = 1.7$ GeV, $T_c = 0.27$ GeV, $P(\chi; T_c) = P(\Phi; T_c)$

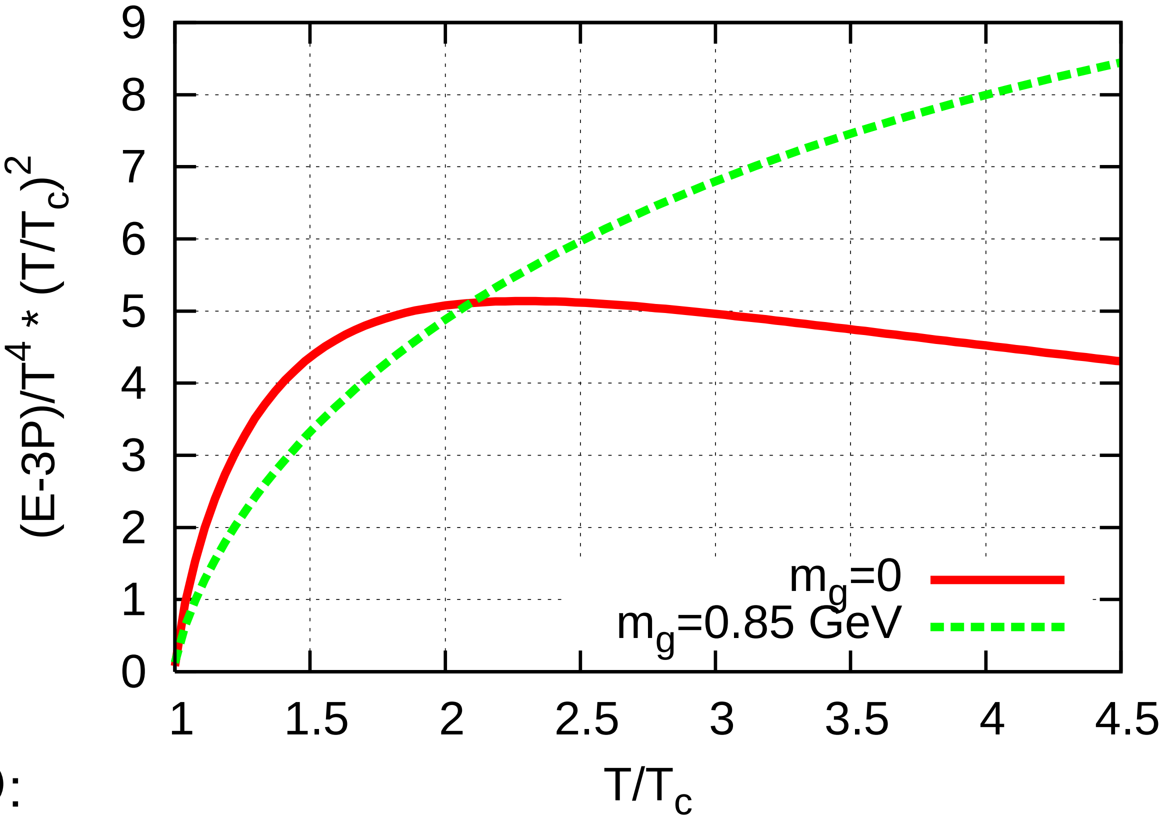
$$B = (0.368\,\text{GeV})^4\,, \quad \chi_0 = 0.16\,\text{GeV}$$
$$a_0 = (0.197\,\text{GeV})^3\,, \quad c_0 = -(0.180\,\text{GeV})^4$$





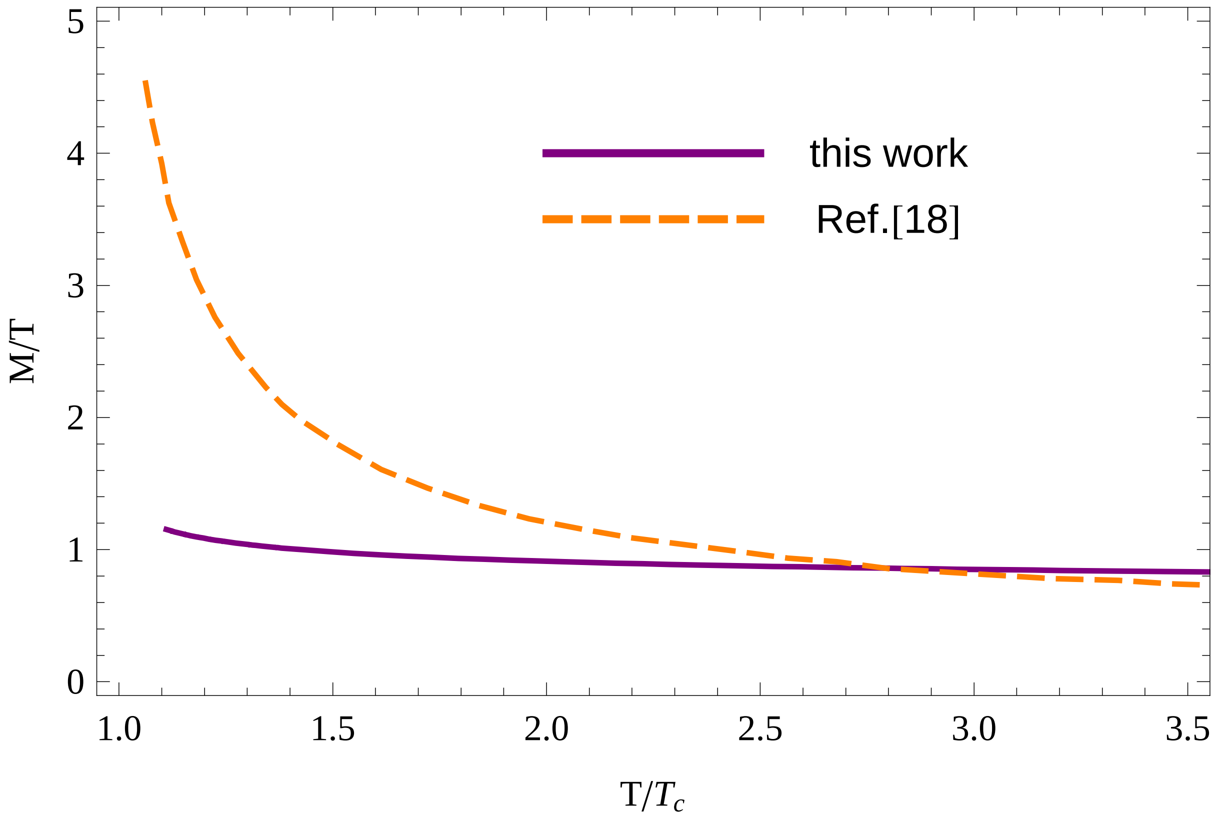
IV. Issues

Interaction measure: massive gluon?



- Lattice QCD:
  - $I(T)/T^4 \sim A/T^2 + B/T^3 + C/T^4$ [Borsanyi et al. (12)]
  - $T/T_c < 3\text{-}4$: $I/T^2 \sim$ const. due to "residual interaction"
- model: $E_g^2 = p^2 + M_g^2$
  - massless gluons: $I \sim T^2$, high T?
  - massive gluons: $I \sim T^3$, condensate of dilaton? resummed $g(T)T$?

- effective gluon mass extracted from lattice EoS [Ruggieri et al. (12)]



  – based on quasi-gluon models w/ Polyakov loops vs. w/o Polyakov loops
  – role of Polyakov loop: $M/T \sim \mathcal{O}(1)$!
  – cf. QPM w/o Polyakov loop: $M \gg T_c$

- standard dispersion $E_g = \sqrt{p^2 + M_g^2}$? or modified in hot medium?

  $\Rightarrow$ non-trivial p-dep. would come in. cf. pQCD

Electric vs. magnetic sector

- chromoelectric sector changes qualitatively with T: electric screening
  deconfinement phase transition driven by electric gluons
- chromomagnetic sector unaffected: no magnetic screening
  string tension non-vanishing at any T $\Rightarrow$ confined, non-trivial above T_c

- **Polyakov loops vs. dilatons**
  - $\Phi \Leftrightarrow A_0$: electric
  - $\chi \Leftrightarrow G_{\mu\nu}G^{\mu\nu}$: electric plus magnetic
  - effective theory above T_c: Φ and χ

$$\mathcal{L} = \mathcal{L}_\Phi + \mathcal{L}_\chi + \underbrace{\mathcal{L}_{\text{mix}}}_{\sim\chi^4\left(a\bar{\Phi}\Phi+b\left(\bar{\Phi}^3+\Phi^3\right)+\cdots\right)}$$

  - effective gluon mass? $\sim G^2\chi^2 A_\mu A^\mu$ would appear when hard modes integrated out $\cdots$ $m_g \sim \langle B^2\rangle$ in deconfined phase

how to transmute $\langle E^2\rangle$-$\langle B^2\rangle$ interaction into Φ-χ interaction?

How to introduce quarks?

- pert. part $(T < T_c, \Phi \sim 0)$

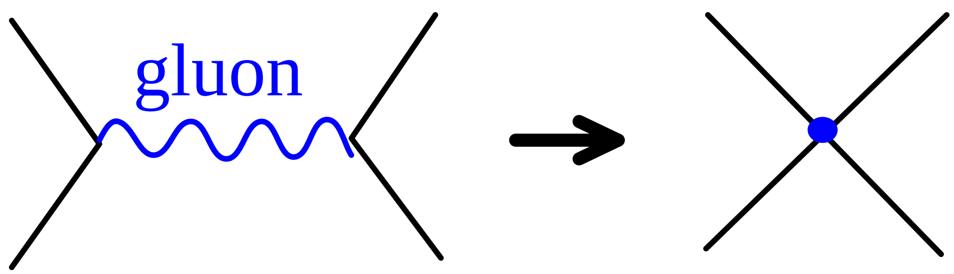
$$\Omega_{g+q} \sim 2T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-E_g/T}\right) - 4N_f T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + e^{-3E_q/T}\right)$$

$$\sim \frac{T^2}{\pi^2}\left[M_g^2 K_2\left(\frac{M_g}{T}\right) - \frac{2N_f}{3} K_2\left(\frac{3M_q}{T}\right)\right]$$

with effective masses: $M_g \equiv M_{\rm gluball}/2\,, \quad M_q \equiv M_{\rm nucleon}/3$

therefore $s = -\partial\Omega/\partial T \overset{?}{\sim} -\partial\Omega_{g+q}/\partial T < 0$

- non-pert. part $\sim$ **effective 4-fermi interaction**



$\cdots$ non-local interaction: T, Φ, p $\Rightarrow$ crossover at $\mu = 0$

- interaction to dilatons:
  interplay between chiral and scale symmetry breaking

Summary

- **role of Polyakov loops in quasi-particle approaches**
- **derivation of gluon partition function from YM Lagrangian**
  - Polyakov loops naturally appear representing group character.
  - gluons are forbidden below T_c dynamically.
  - a hybrid approach.
- **higher representations of Polyakov loop**
  - non-vanishing even in confined phase: mean field artifact
  - do not condense when energy distributions are expressed in fund. rep.

- **Polyakov-loop susceptibilities vs. LQCD** [Lo-Friman-Kaczmarek-Redlich-CS]
- **effective theory of magnetic sector** [CS-Mishustin-Redlich]
- **introducing quarks**